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A Historical Perspective on Quantum Physics and its Impact on Society

An Interactive Qualifying Project Report:

Submitted to the Faculty

of the

WORCESTER POLYTECHNIC INSTITUTE

In partial fulfillment of the requirements for the

Degree of Bachelor of Science

By

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Date: June 1, 2009

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Abstract

The goal of this IQP is to investigate the problems of physics that led to the birth of quantum physics, the scientists that were involved in its development, and the impact quantum physics has had on society through some of its applications.

1 Introduction

Quantum theory is a theory on which most of our understanding of the material universe is based. It is a relatively new theory which was begun in the early 20th century. By the end of the 19th century, physics consisted mainly of Newton's classical laws of motion and Maxwell's electromagnetic theory. Newton's law of mechanics was used to describe the dynamics of macroscopic objects and Maxwell's theory of electromagnetism was used to describe radiation. Light was believed to be an electromagnetic wave that obeyed Maxwell's law of electromagnetic radiation. At that time the overwhelming success of classical mechanics made physicists believe that the ultimate description of nature had been established. However, as soon as new experimental techniques were developed that allowed physicists to observe matter at the atomic and subatomic level, classical physics failed miserably in providing an explanation for the newly discovered phenomena. This showed that classical mechanics which worked well at the macroscopic level was inadequate in describing the material universe at the microscopic level and new concepts were needed to describe this microscopic world.

In 1905, through the photoelectric effect, Einstein showed that light had properties that were proper to particles. This was followed by Bohr's model of the atom in 1913 which was developed as an improvement on Rutherford's planetary model and which showed that the dynamics of microscopic particles like electrons were governed by laws different from the classical laws of Newton. Then Louis De Broglie made the bold suggestion that all particles have waves like characteristics. In 1926 Heisenberg and Schrödinger gave a precise formulation of De

Broglie's ideas through the theory of quantum mechanics. This theory brought about a complete resolution to the problems by showing the wave aspect of quantum mechanics.

Since its birth, quantum mechanics has proved to be a very successful theory of physics. It is believed to be the most fundamental theory of physics; many physicists believe that all physical and chemical phenomena are derivable from its postulates and laws. Its validity has been checked against the operation of nature and found to be correct. Quantum physics has helped physicists gain a great insight into physical phenomena and to make remarkable predictions about the outcome of experiments. However, quantum theory is still a model of the universe and as such is just an approximation (although a very accurate one so far) of physical reality which will be revised as our experience grows.

Since its birth, quantum physics has had a huge impact on society through its numerous applications in electronics, medicine, military, etc.... Much of today's technology operates at a scale where quantum effects are important. Some popular applications of quantum physics include Lasers, X-rays, holograms, semi-conductors, transistors, MRI (magnetic image resonance), and quantum computers.

In this paper we will first revise the physical ideas and experimental facts observed at the end of the 19th century and the beginning of the 20th century that defied classical physics and led to the birth of quantum physics. We will then consider a couple of applications of quantum physics that are critical to the operation of today's society.

2 Origins of quantum physics

2.1 BLACKBODY RADIATION

Early in the 1900s, physicists became interested in the study of blackbodies to know how they emit and absorb radiation. Radiation is energy in the form of waves of moving subatomic particles emitted by an atom or another body as it changes from a high to a low energy state. A blackbody is a material that absorbs all incoming radiation and does not reflect any.

Around 1860, the German physicist, Robert Kirchhoff, proposed the idea of blackbody radiation. He noticed that the energy emitted by the body is independent of its geometry and chemical composition but was only dependent on its temperature. Then came along the Austrian physicist Josef Stefan who suggested in the year 1879 that heat radiated from Kirchhoff's blackbody varied with the fourth power of the blackbody's absolute temperature. Five years later, another Austrian physicist, Ludwig Boltzmann, proved this suggestion by combining Maxwell's electrodynamics with the second law of thermodynamics. He obtained the following relation:

$$u = \sigma T^4.$$

Here u is the total energy density of the blackbody, T its absolute temperature, and σ is the Stefan-Boltzmann constant.

With the establishment of the Stefan-Boltzmann law, more and more physicists became interested with the development of the theoretical and experimental results related to the blackbody. It did not take long for physicists to be interested in resolving the spectral distribution of the blackbody. The spectral distribution of a blackbody describes how the body radiates heat at different wavelengths and at a fixed temperature. Physicists knew that hot objects radiate heat at different wavelengths of light depending on the temperature.

The English physicist, Lord Rayleigh, was one of the first to derive an explicit formula to describe the spectral distribution of a blackbody. That formula, known as the Rayleigh-Jeans law, predicted that the blackbody will emit radiation with infinite power as the wavelength goes to zero. This behavior disagreed with experimental results obtained at the time, which showed that the radiation power remained bounded at short wavelengths. Rayleigh-Jean's law agreed with experimental data at large wavelengths but did not fit it at short wavelengths. This disagreement became known as the ultraviolet catastrophe.

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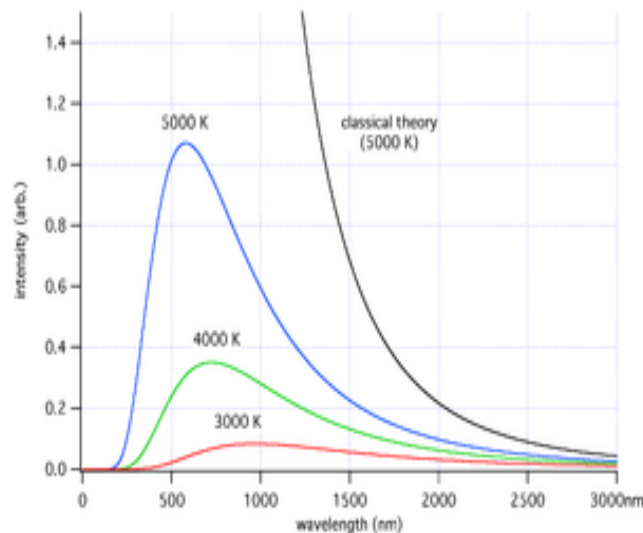


Figure 2-1: Ultraviolet catastrophe.

An important step toward the solution was provided by Wilhelm Wien who in 1884 showed that if the spectral distribution of the blackbody is known at one temperature then it is known at other temperatures. This showed that the distribution function did not depend separately on the temperature T and the wavelength λ ; instead it depended on the product λT through some function $u(\lambda, T) = \lambda^5 \phi(\lambda T)$. Wien's displacement law – so called because it implies that the peak of the $u(\lambda, T)$ function will be displaced toward smaller wavelengths when T increases – was found to agree excellently with experiments. The importance of the function

$\phi(\lambda T)$ was recognized universally, even though there was no theoretical justification for its form. In the year 1896, Wien became the first physicist to guess a possible form of the function $\phi(\lambda T)$; According to him the function $\phi(\lambda T)$ was of the form $e^{-\alpha/\lambda T}$, with α a universal constant. Wien's radiation law was commonly accepted among physicists due to its agreement with a series of experiments carried out in Berlin between 1897 and 1899.

Even though Wien's law appeared to be correct, it still lacked a theoretical foundation. Wien's had justified his law using arguments of an unsatisfactory nature; hence a more rigorous derivation was needed. The German physicist Ludwig Planck, who was Kirchhoff's successor at the time as professor of physics at the University of Berlin, was the first to provide a derivation of Wien's law from first principles.

Planck was both a specialist of physics and thermodynamics. He was interested in the application of the second law of thermodynamics to problems in physics and chemistry. In his work the concepts of entropy and irreversibility were central. Planck, unlike many physicists at the time who believed that the concept of irreversibility was probabilistic in nature, firmly believed that it could entirely be explained on a thermodynamics basis.

However, his thermodynamics approach proved to be unsuccessful in the derivation of the spectrum of the blackbody; hence Planck had to reconsider a new approach. His renewed effort resulted in the publication of a series of six papers published in *Annalen der Physik*, between the years 1897 and 1900, on the topic of irreversible radiation processes. In one of these papers, he had found an expression of the entropy of an oscillator from which he could derive Wien's radiation law. Planck had reached his goal and was it not for experiments he might have stopped there.

A series of experiments carried out the same year Planck derived Wien's law showed that Wien's law was not completely correct as many physicists had assumed at the time. It is important to note that at the time the determination of the precise spectrum of the blackbody was a matter of more than academic interest. It was believed at the time that the analysis of the spectrum of the blackbody would produce physical knowledge that would be useful to the German heating and lighting companies, which were amongst the biggest customer of some of Germany's largest experimental laboratories such as the *Physikalisch-Technische Reichsanstalt* (Imperial Institute of Physics and Technology).

Detailed experimental work by Lummer and Pringsheim (1899) and Rubens and Kurlbaum (1900) had given definite proof that Wien law was incorrect for long wavelengths and was only approximately true for other ranges of the wavelength. Wien's law had predicted that the radiation energy density would tend to zero at large wavelengths while the experiment had shown otherwise. This inconsistency between experiments and Wien's law was followed by the proposal of many new empirically based laws.

However the theorists were still interested in a law derived from first principles. Planck was forced to reconsider his work. He needed to first understand what went wrong in the derivation of the Wien's law. In his attempt to fit the experimental data, Planck guessed, without theoretical justification, a new expression for the entropy of an oscillator. His assumption was in complete agreement with the experiment. About this expression it has been remarked, "Never in the history of physics was there such an inconspicuous mathematical interpolation with such far reaching physical and philosophical consequence" (Jammer 1966). With his new expression for entropy, Planck was able to derive an improved version of Wien's law which was in agreement with the experiment over the whole spectrum.

According to the new law, the spectral energy density varied as ν^3 divided by the quantity $\exp(\beta\nu/T)-1$. The success of the new formula which was based on an inspired guess of the entropy expression was not too satisfactory for Planck; He wanted a theoretical explanation as to why the new law worked. This led Planck to reconsider Boltzmann's probabilistic view of entropy as an expression of molecular chaos, and to interpret it in his own non probabilistic way. Starting from Boltzmann's equation, Planck assumed that the energy of the oscillator of the blackbody was subdivided into finite portions ϵ , called energy elements (energy quanta). Each energy quanta had an energy equal to $h\nu$, where $h=6.55 \cdot 10^{-27}$ is the universal constant which later became known as Planck's constant, and ν the frequency of the oscillator. This assumption that the energy of the oscillator was divided into discrete packages marked the birth of the quantum hypothesis.

Planck presented his result at the Berlin academy on December the 14th, 1900. This date is considered by many to be the birthday of quantum physics. In what followed, Planck law became widely used due to its perfect agreement with experiment. Planck himself did not fully understand why the energy of the oscillator was divided into discrete packages. The discreteness assumption was simply a mathematical hypothesis with no real physical reality behind it, and at first Planck and his contemporaries did not pay serious attention to the law nor did they recognize that it necessitated a break from classical physics. What mattered was rather the impressive accuracy of the new radiation law, confirmed by many experiments.



Figure 2-2: Max Planck

It is important to note that Planck went on to win the Nobel Prize in physics in the year 1918, for his work on blackbody radiation. He is regarded by many as the father of quantum physics.

2.2 Photoelectric and Compton Effect

The photoelectric effect is a phenomenon that was discovered by the German physicist Heinrich Hertz in 1887. At the time it was an observation that could not be explained by the classical theory of radiation (Maxwell's theory of radiation). Light shun on a metal was observe to eject electrons only above a certain threshold frequency that was proper to the metal, and when light with frequency above the threshold value was shun on the metal, electrons were ejected instantly regardless of the magnitude of the intensity of the light. In addition, the number of electrons ejected from the metal was observed to depend on the intensity of the light and not its frequency. Furthermore, the kinetic energy of the ejected electrons increased linearly with respect to the frequency of the light and did not depend on the intensity of the light. According to the classical electromagnetic theory of radiation established at the time, electromagnetic waves like light could exchange any amount of energy with matter and since the energy of an electromagnetic wave monotonically depends on its intensity, light of any frequency with a high enough intensity would provide the energy necessary to free the electron from the metal.

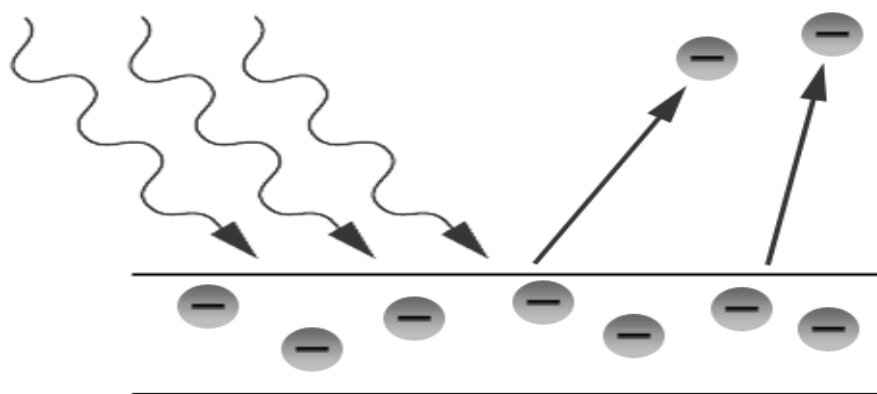


Figure 2-3: Photoelectric Effect

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Also, the classical electromagnetic theory predicted that light of a very low intensity would take some time to free electrons from the metal, regardless of its frequency. All these predictions of the classical electromagnetic theory were in direct opposition to the observations of the photoelectric effect. These experimental facts showed that the classical concept of gradual absorption of energy by the electrons as predicted by quantum physics was wrong.

In 1905, inspired by the ideas of Planck, the German physicist Albert Einstein was able to give a theoretical explanation of the photoelectric effect. His point of departure was the radical assumption that light is composed of particles called photons each having an energy E proportional to the frequency of light and given by

$$E = hf$$

Where h denotes Planck's constant introduced in the previous section. When a beam of light is incident on the surface of a metal, a photon of energy hf will be absorbed by an electron and in the process will transmit all its energy to the electron. This suggested that electrons could only absorb the energy of light in discrete energy quanta regardless of the intensity of the incident

radiation. The minimum energy W required to free an electron from a specific metal is called its work function and the condition for photoelectric emission is $W < hf$. This condition determines a threshold frequency $f_0 = \frac{W}{h}$ and the kinetic energy of the escaping electron is then given by the equation

$$hf = hf_0 + K.$$

This equation clearly shows the linear dependence of the kinetic energy with respect to the frequency and shows why no electron can be ejected from the metal if $f < f_0$.

In 1916, the American physicist Robert Andrews Millikan gave a systematic confirmation of Einstein's photoelectric effect. Using various metals for his experiment, he was able to show that the energy of the photoelectron increases linearly with increasing frequency of the incident light, and is independent of the intensity of the incident light. From his experimental work he was also able to determine Planck's constant within an accuracy of 5%.

In summary, the photoelectric effect gave evidence to the corpuscular nature of electromagnetic radiation. Einstein went on to receive the Nobel Prize in 1921 for his work on the photoelectric effect.

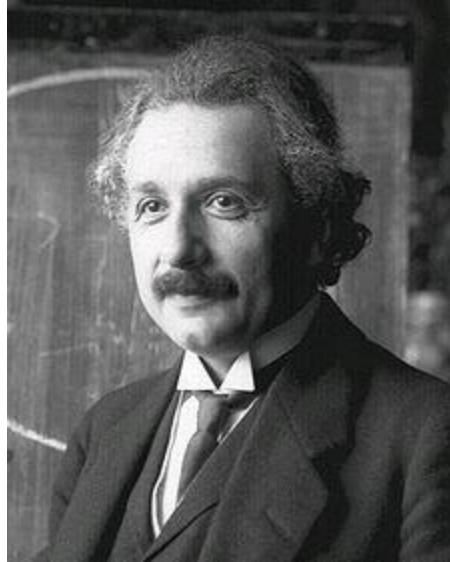


Figure 2-4: Albert Einstein, 1921

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Further convincing evidence of the particle like nature of electromagnetic radiation was provided by the Compton Effect. The Compton Effect was observed in 1923 by the American physicist Arthur Holly Compton by scattering X-rays off free electrons. He found that the wavelength of the scattered X-ray was smaller than that of the incident X-ray, indicating an inelastic interaction between the X-ray and the electron. The classical theory of electromagnetic radiation predicted that the incident and scattered wave should have the same wavelength. To explain his experimental results, Compton treated the incident X-ray as being composed of a stream of photons interacting elastically with individual electrons.

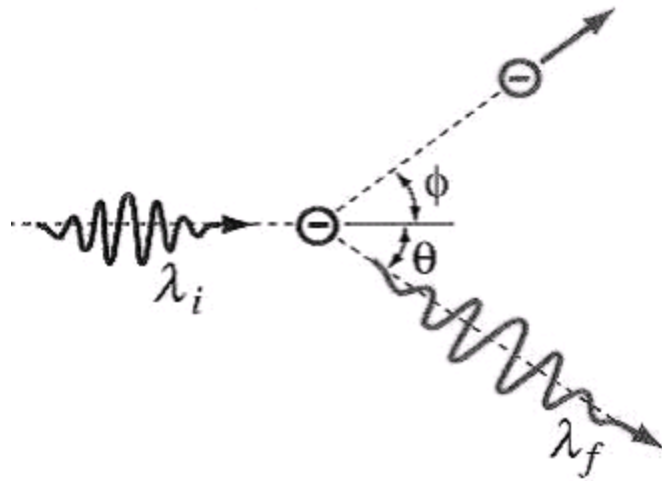


Figure 2-5: Compton Effect

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In summary the Compton Effect gave further evidence that photons behave like particles.

Compton went on to receive the Nobel Prize for his work on the Compton Effect in 1927.

Another phenomenon like the photoelectric and Compton Effect that gives evidence of the corpuscular nature of electromagnetic radiation is the process of pair production. This process is a relativistic process and can be explained by relativistic quantum mechanics (a combination of Einstein theory of relativity with Schrodinger quantum mechanics). It is a process through which a photon is a process through which an electron is annihilated to produce a particle and an antiparticle pair. When photons from a high energy electromagnetic radiation interact with the electrons of a metal, they disappear producing an electron and a positron. A positron is an antiparticle to an electron that has the same mass as an electron but an equal but opposite charge. The existence of positrons was conjectured by the British physicist Paul Dirac, and they were first discovered experimentally by Anderson.

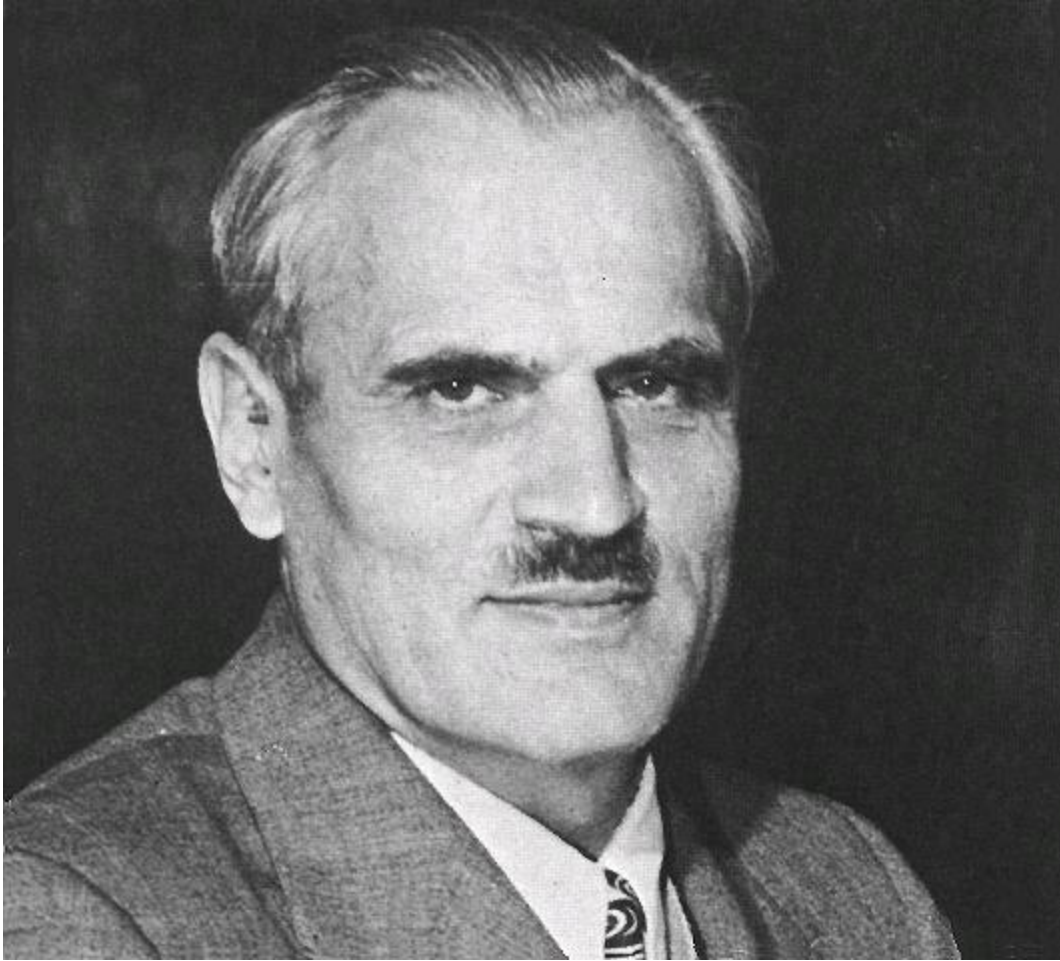


Figure 2-6: Arthur Holly Compton

2.3 Bohr Model of the atom

The “nuclear atom” is a term that refers to the model of the atom in which an atom is composed of a small (radius $\approx 10^{-15}\text{m}$) positively charged nucleus which is surrounded at relatively large distances (radius $\approx 10^{-10}\text{m}$) by orbiting electrons as depicted in Fig. 2-7. This model of the atom is the most universally accepted one and it is quite recent. The nuclear atom was proposed by the New Zealander physicist Ernest Rutherford as an improvement on the “plum-pudding” model of the British physicist J.J. Thomson.

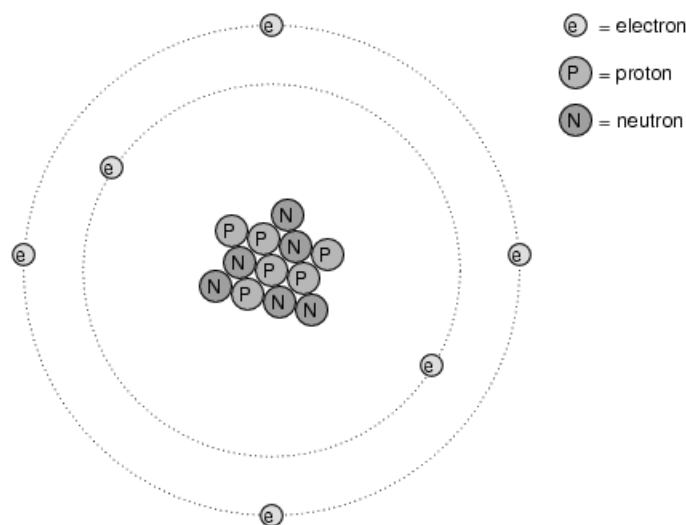


Figure 2-7: Rutherford's Atom

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In the plum-pudding model of J.J Thomson an atom had no nucleus at its core. The positive charge was assumed to be spread throughout the atom forming a kind of “pudding” in which negative electrons were suspended like “plums” as shown in Fig 2-8.

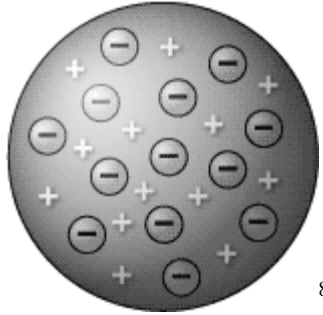


Figure 2-8: Plum-Pudding model

To discredit the plum-pudding model, Rutherford carried out an experiment (now known as Rutherford experiment) in which atoms were observed to scatter alpha particles (nuclei of helium atoms) at relatively large angles which could not be explained by the plum-pudding model. The plum-pudding model by assuming a uniformly distributed positive charge throughout the atom would have predicted no deflection of the alpha particles by the atom. The fact that deflections were observed could only mean that the positive charge was concentrated in a very small region of the atom. In order to counteract the electrostatic attraction between the positive nucleus and the negative electrons, Rutherford determined that electrons must be moving around the nucleus like planets revolving around the sun. This is why the nuclear model is often referred to as the planetary model.

The planetary model of the atom despite being successes was faced with one major obstacle: it predicted that even light atoms like hydrogen were unstable. According to classical electromagnetic theory, an electron revolving around a nucleus will radiate off electromagnetic waves which will deplete the electrons energy and cause it to spiral inward toward the nucleus (see Fig. 2-9). As a conclusion, all atoms will be unstable which is contrary to the observed stability of light atoms.

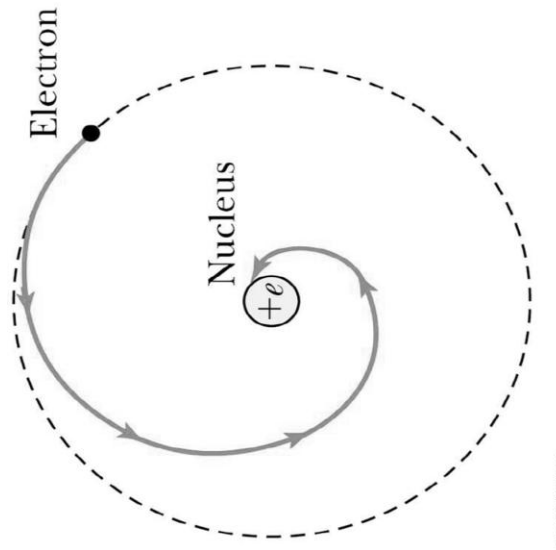


Figure: 2-9: An electron falling into the nucleus as it radiates off its energy.

9

In addition, the classical theory predicted that as the electron fell into the nucleus, it would radiate at higher frequencies as the orbit of the electron got smaller and faster. This predicted a continuous emission spectrum for the hydrogen atom contrary to the empirically observed discrete one. Three empirical formulas were developed at the time which reproduced the values of the observed wavelengths of the line spectrum of hydrogen in three mutually exclusive regions of its spectrum. These empirical formulas were known as the Balmer series, the Lyman series and the Paschen series

In 1913, the Danish physicist Neil Bohr presented a model that allowed him to reproduce the equations in the Balmer, Lyman and Paschen series. His point of departure was Rutherford's model of the atom. In addition he made assumptions that incorporated the newly successful quantum ideas of Einstein and Planck.

In order to obtain a discrete set of stable orbits, Bohr postulated, without any explanation that electrons are confined to certain stationary states (orbitals), each having a circular orbit and

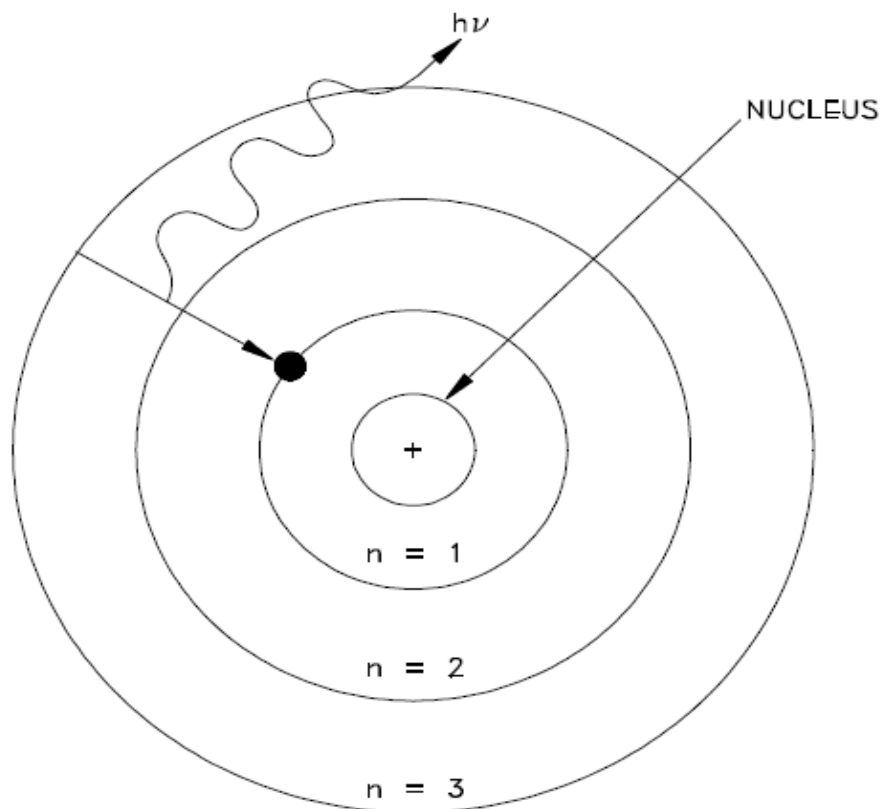
electrons could only radiate energy while transiting from one stationary state to another (this requires electrons to preserve their energy while in orbitals which violates the classical laws of physics). To incorporate Einstein's photon's concept, Bohr theorized that electrons would emit a photon while transiting from a higher energy level to a lower one. The energy of the emitted photon will then be equal to the absolute value of the difference of the energies of the two stationary orbits. Note that this is similar to Planck assumption of energy being absorbed or emitted by a harmonic oscillator in integer multiples of hf . Electrons could also be excited into higher energy levels by introducing a high voltage.

To derive the equation for the values of the energy of stationary states, Bohr made one more assumption: he quantized the angular momentum of the electron. Only discrete values of the angular momentum were allowed, mainly the ones that were integer multiples of Planck's constant divided by 2π :

$$L = \frac{nh}{2\pi}$$

Where n is an integer. De Broglie was later able to give a justification for this assumption using ideas from his own theory on the wavelength of moving particles. With this assumption Bohr was able to derive the values of all possible orbitals radii and then the energy of all orbitals:

$$r_n = (5.29 * 10^{-11})n^2/Z$$



10

Figure 2-10: Bohr's model of the hydrogen atom.

$$E_n = - \frac{(2.18 \times 10^{-18} \text{ J}) Z^2}{n^2}$$

These values of the orbital energies were then combined with the formula for the energy of emitted photons

$$hf = E_i - E_f$$

To reproduce the wavelengths observed on the line spectrum of hydrogen. The agreement between the theoretical and experimental values of the Rydberg constant appearing in the Lyman, Balmer and Paschen series was a major accomplishment of Bohr's theory. Bohr's theory could also be used to accurately predict the ionization energy of atoms - this is the minimum energy that is needed to remove an outer electron from the atom - and ions having one single electron.

These transitions between energy states were considered by Bohr as occurring instantaneously. They are referred to as quantum jumps or quantum leaps. The timing of these jumps and the direction of emitted electrons cannot be predicted; only their probability of occurrence can be computed. An analysis similar to the one carried out by Bohr was developed by Sommerfeld and Wilson in which the circular orbits of Bohr were replaced by elliptical ones.

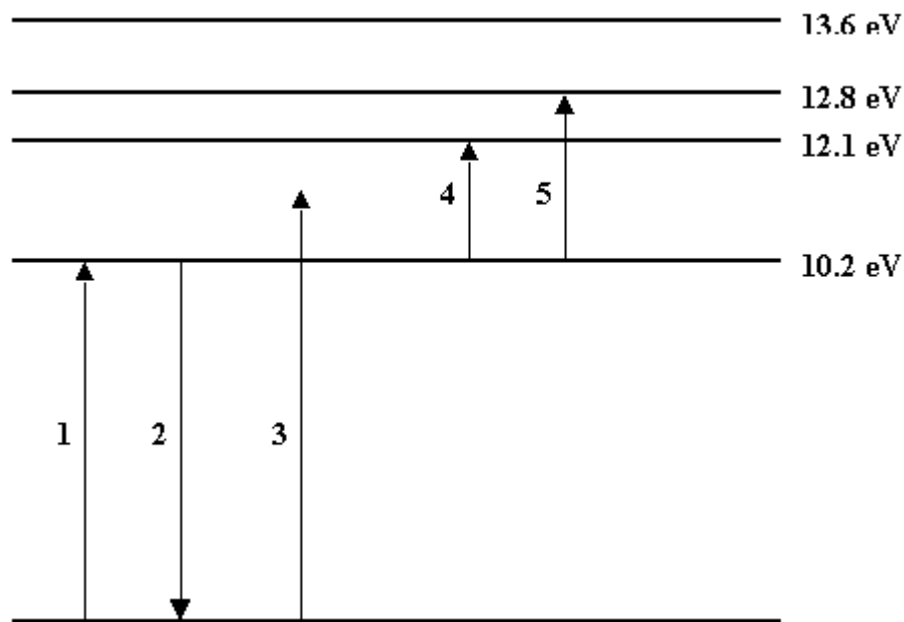


Figure: 2-11: Energy level diagram for the hydrogen atom.

11

Overall, even though the Bohr model was successful in reproducing the line spectrum of the hydrogen atom, it had its shortcomings. One major restriction on the model is that it can only be applied to single electron atoms (hydrogen) or ions. Also the model is not correct as it violates Eisenberg uncertainty principal by assigning a fixed position and momentum to the electron simultaneously. Furthermore it fails to explain why some spectral lines are brighter than others. Despite these shortcomings, Bohr's theory can still be used in more complex settings to make accurate predictions about the output of experiments. For his work on the hydrogen atom Niels Bohr was awarded the Nobel Prize of physics in 1922.



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Figure 2-12: Niels Bohr

2.4 WAVE-PARTICLE DUALITY

The Wave-Particle Duality principle of quantum physics states that all matter and energy in nature can behave both as a wave or a particle depending on the conditions of the experiment. By the end of the 19th century, physics consisted mainly of Newton's classical laws of motion and Maxwell's electromagnetic theory. Newton's law of mechanics was used to describe the dynamics of macroscopic objects and Maxwell's theory of electromagnetism was used to describe radiation. In this classical context, waves and particles were mutually exclusive entities: particle could be completely specified by their position vector and waves by their phase and amplitude.

The concept of duality emerged out of the debate on the nature of light. Was light made of waves or particles? During the 1600s, the English physicist Sir Isaac Newton proposed a corpuscular theory of light which could explain phenomena such as reflection, refraction and the splitting of light by a prism. During the same period, the Dutch physicist Christian Huygens proposed a new theory of light which treated light as a wave instead of a particle. However there were many observations made at the time that Huygens theory, still in its infancy, could not solve. This inability of Huygens theory to explain fundamental observations, coupled with Newton's fame, led the scientific community to adopt Newton's corpuscular theory of light, which remained dominant in the following century. However, the belief in the validity of Newton's theory all changed around the 18th century. The first observation that invalidated Newton's theory came in the form of an experiment carried out by the English physicist Thomas Young and the French physicist Augustin-Jean Fresnel, now known as the "double slit experiment", which showed that light shun through a small slit displayed interference pattern

proper to waves (see Fig 2-13). From the experiment they were able to determine the wavelength of light. Their theory also allowed them to explain phenomena such as polarization that the corpuscular theory of Newton could not. Such evidence lent strong support for the wave theory of light, which became dominant throughout the 1800's. The popularity of the theory even grew further as the Scottish physicist James Clerk Maxwell developed a set of equations - now known as Maxwell's equations - which explained the propagation of light as that of electromagnetic waves.

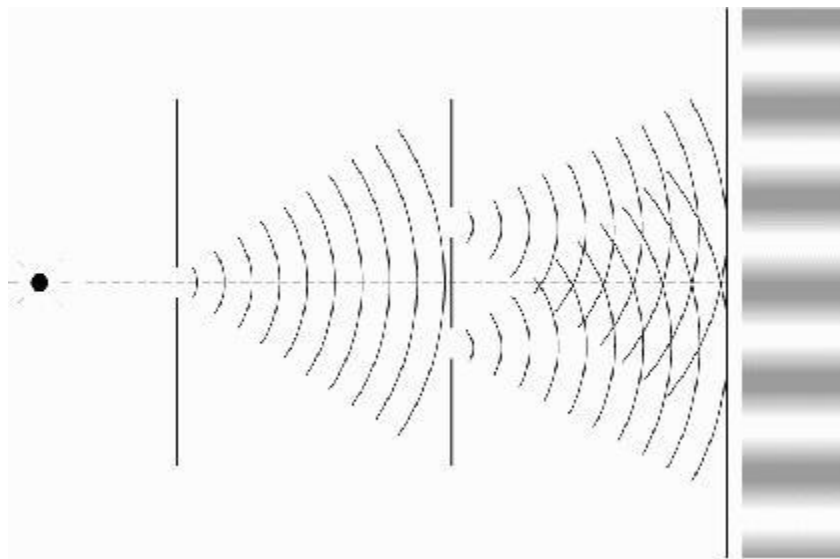


Figure: 2-13: Double slit experiment ¹³

The ability of the wave theory to explain recent experimental observations made it quite popular among physicists during the 19th century. However, by the turn of the 20th century, new observations were being made that were at odd with the theory. One such observation was the phenomena of blackbody radiation, which showed that the spectrum of electromagnetic radiation emitted by a blackbody was not continuous but discrete. The German physicist Planck solved the problem of blackbody radiation by assuming that the energy of the atoms of the blackbody had a discrete spectrum. This was in opposition with the wave theory of the time which predicted a continuous spectrum for the atoms of the blackbody. Around the same time period experiments

carried out by the German Physicist Albert Einstein showed that light had particle like properties. Einstein's experiment (see Fig 2-14) -now known as the Photoelectric effect - showed that by shining light on a metal, the energy carried by light could knock off electrons in the metal and generate an electric current. The wave theory of light predicted that a brighter light source would generate a higher current – due to its higher intensity - than a dimmer one. However the experiment showed that dim blue light generated a higher current than the stronger red light, which released no electrons at all. Einstein was able to resolve the conundrum by postulating a quantification of the electromagnetic radiation: Electromagnetic radiations (light) carried energy in discrete packages called photons (light was not a continuous wave as assumed by Maxwell's theory), and the energy of a photon was directly proportional to its frequency, the constant of proportionality being Planck's constant

$$E = hf$$

Where $h=6.626 \times 10^{-34}$ J seconds. Hence blue light due to its high frequency (above the threshold frequency of the experimental metal) carried photons with high enough energy to knock off electrons while red light with its low frequency (below the threshold frequency) carried photons of low energy.

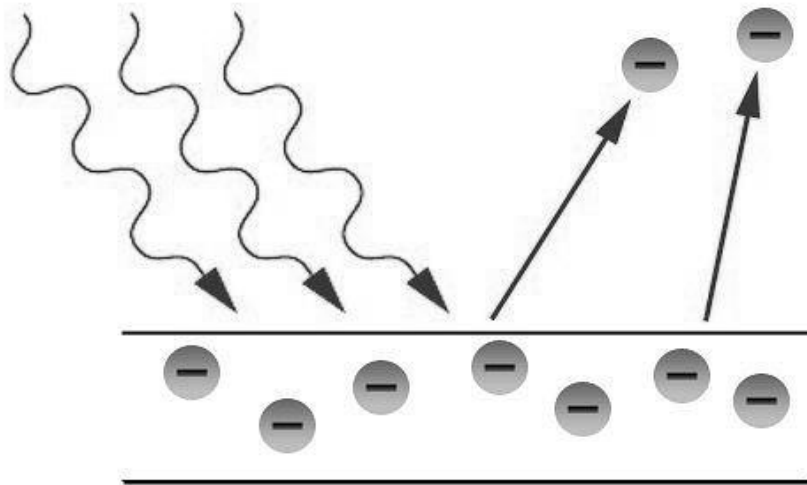


Figure: 2-14: Photoelectric effect

All these observations led the scientific community during the early years of the 20th century, to reconsider Maxwell's theory of light which was standard at the time. A dual theory of light seemed more appropriate than a corpuscular or wave theory alone.

Inspired by the wave-particle picture of light and by the success of the Bohr model (the Bohr model will be described in the next chapter) the French physicist Louis Victor De Broglie made a radical proposal. If light waves with energy $E = hf$ under certain circumstances behaved like particles of energy E , then by symmetry a massive particle of energy $E=pc$ (as given by Einstein theory of relativity) should, under some circumstances, behave like a wave with energy E . By equating the formulas for the energy, De Broglie was able to derive an expression for the frequency of a particle of energy E and momentum p :

$$f = \frac{E}{h}$$

And a similar expression for its wavelength:

$$\lambda = \frac{h}{p}$$

Where h is Planck's constant and p the momentum of the particle. This was an extension of Einstein's equation, since it reduced to the equation for the energy of a photon upon substitution of the momentum of an electromagnetic radiation. The validity of De Broglie's formula for electrons was confirmed in the following years by two experiments during which electron diffraction was observed. One of these experiments was carried out by the English physicists Sir George Paget Thomson (Son of the Nobel Laureate J.J. Thomson) and Clinton Joseph Davisson. The experiment showed that by passing an electron beam through a thin metal film an interference pattern proper to waves could be observed on the screen (see Fig. 2-15). Similar successful experiments were carried out much later for heavier sub particles like protons and neutrons.

The hypothesis of De Broglie was later used by the Dutch physicist Ernest Schrodinger to derive the Schrodinger wave equation and subsequently lead to the development of quantum mechanics as a theory describing the atomic world.

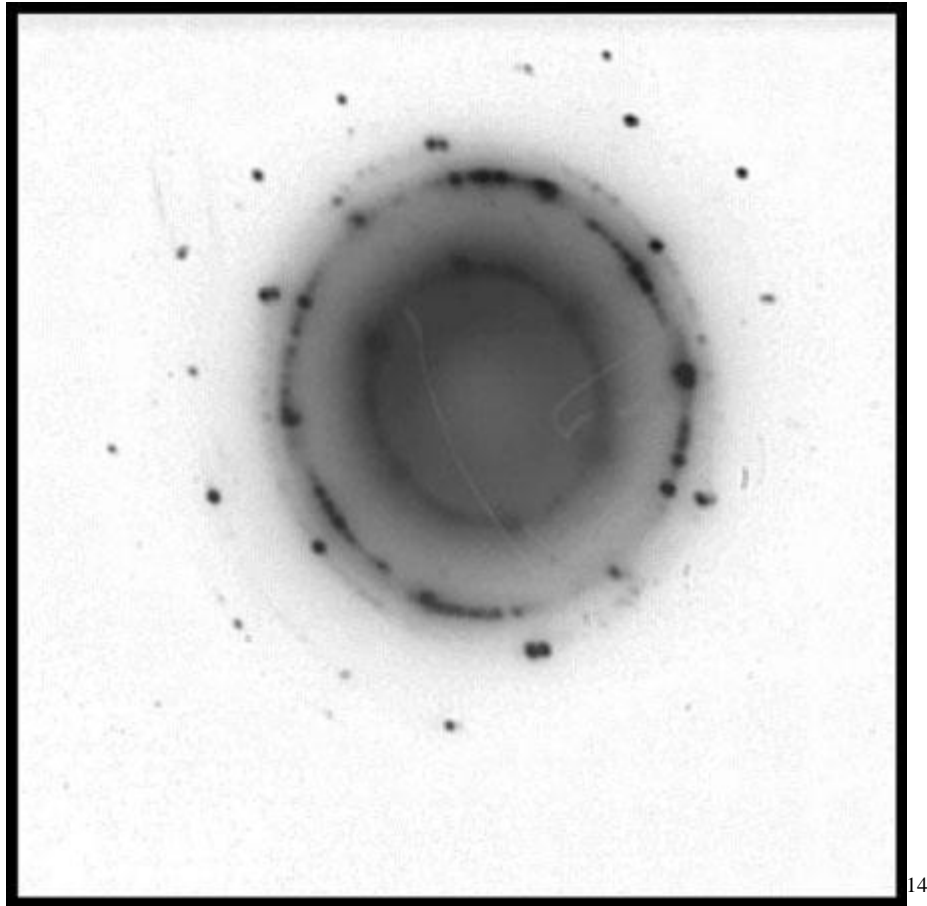


Figure 2-15: Electron diffraction pattern

The wave-particle duality principle, even though a very difficult concept to grasp, is a well-accepted concept amongst physicists nowadays. It allows physicists to explain the behavior of light and matter through differential equations of the Schrödinger type. Through its use physicists have been able to make very accurate and non intuitive predictions to the outcome of experiments. Even though many interpretations of the concept exist among physicists, the meaning of the concept itself is still a question of debate among quantum physicists.

De Broglie was awarded the Nobel Prize of physics in 1929 for his theoretical work on De Broglie's hypothesis and Thomson and Davidson were awarded the 1937 Nobel Prize in physics for their experimental work on electron diffraction.



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Figure 2-16: Louis De Broglie

2.5 Pauli Exclusion Principle

There was a need among physicists at the beginning of the 20th century for a theory that could explain how sub-particles – protons, neutrons, and electrons – combine to form atoms. Such a theory would explain the structure of the periodic table. Around the same time, some observations on the stability of certain atoms were being made that physicists at the time could not explain: atoms with an even number of electrons were observed to be more stable than those with an odd number of electrons

In 1925 the Austrian physicist Wolfgang Pauli formulated the Pauli Exclusion Principle which gave a rule for determining the electronic configuration of an atom in its ground state. The ground state is the state of the atom at which it attains its lowest energy. The electrons of an atom at room temperature spend most of their time at the ground state. He first introduced four parameters – quantum numbers – that could be combined to describe the configuration of an electron in an atom. These four quantum numbers were:

- The principal quantum number n determines the total energy of the electron in the atom and is only allowed to take integer values

$$n = 1, 2, 3 \dots$$

- The orbital quantum number l , which is allowed to take the values

$$l = 0, 1, 2, \dots, n - 1.$$

The magnitude L of the angular momentum of the electron is

$$L = \frac{\sqrt{l(l+1)}h}{2}$$

- The magnetic quantum number m_l describes the effect of an externally applied magnetic field on the energy (its value is zero when no external magnetic field is present) and is allowed to take the values

$$m_l = -l, \dots, -2, -1, 0, 1, 2, \dots, l.$$

The component L_z of the angular momentum in the direction of the applied magnetic field is directly proportional to the magnetic number

$$L_z = m_l \frac{h}{2\pi}.$$

- The spin quantum number m_s , which gives a direction to the angular momentum of the electron and can only take the values

$$m_s = 1/2 \text{ or } m_s = -1/2.$$

The state of all electrons in an atom can be determined by an admissible combination of four quantum numbers. Electrons that have the same principal quantum number n are said to be in the same shell and those that have the same principal quantum number n and orbital quantum number l are said to be in the same subshell. The energy level of each state of a multi electron atom depends both on its principal quantum number n and its orbital quantum number l and increases as both n and l increase. Pauli's Exclusion Principle states that in an atom no two electrons can have the same set of values for the quantum numbers n , l , m_l , and m_s . Pauli Exclusion Principle explained the structure of the periodic table and hence the chemical behavior of atoms.

Wolfgang Pauli was awarded the Nobel Prize in physics in 1945 for his discovery of the Exclusion Principle.



Figure 2-17: Wolfgang Pauli

2.6 Quantum mechanics

Building on the work of Planck, Einstein, Bohr, De Broglie, and many others, the German physicist Werner Heisenberg in 1925 and the Austrian physicist Erwin Schrodinger in 1926 developed the theory of quantum mechanics in an attempt to reconcile the wave and particle aspect of matter. In the new theory, two new objects were of central importance: operators and wave functions. Operators were used to describe the measurable properties of matter – position, momentum, energy, etc – and the wave function was used to describe the quantum state of microscopic systems taking into account both their particle and wave characteristics. In the Heisenberg formulation of quantum mechanics, vectors were used to describe the state of microscopic systems and matrices were used to describe measurable properties. It became known as matrix mechanics for its use of the mathematical theory of matrices that was not very popular at the time among physicists. Heisenberg's formulation on the other hand used waves which were familiar to physicists at the time. Schrodinger's formulation became known as wave mechanics. The British physicist Paul Dirac later showed that the two formulations were equivalent and developed a third abstract mathematical formulation of quantum physics.



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Figure 2-18: Werner Heisenberg

In classical physics, the concept of wave and particle preclude each other but in quantum physics they do not contradict or preclude one another but as suggested by Bohr they are complementary. The fact that waves are used to describe microscopic particles implies that they cannot be localized in space, since waves are spread out over some region in space. The classical concepts of exact position, exact momentum and exact energy therefore make no sense at the microscopic scale and a probabilistic description of quantum systems should be used. This indeterministic nature of the microphysical world is the essence of Heisenberg's uncertainty principle. To observe with precision the position of a microscopic particle like an electron, one needs to use radiation of very short wavelength (about the size of an electron). Such radiation due to its short wavelength carries a lot of energy and changes considerably the momentum of the electron – enough that it can even knock it out of its orbit - upon incidence thus disturbing its quantum state.



Figure 2-19: Erwin Schrodinger

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In its original form, the Heisenberg uncertainty principle states that if the x-component of momentum is known within an accuracy of Δp_x , the x-component of position cannot be known with accuracy greater than $\Delta x = \frac{\hbar}{2\Delta p_x}$. In three dimensions, Heisenberg's uncertainty principle for position and momentum take the form

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}, \quad \Delta p_y \Delta y \geq \frac{\hbar}{2}, \quad \Delta p_z \Delta z \geq \frac{\hbar}{2}.$$

This principle indicates that it is not possible to simultaneously measure the position and momentum of a microscopic particle at an arbitrary accuracy. Its most general form applies to any pair of complementary variables like energy and time for example and in this context takes the form:

$$\Delta E \Delta t \geq \hbar/2$$

This equation is very useful in the study of decay processes since it gives the relationship between the mean lifetime Δt and the energy width of the excited state ΔE . It explains the fact that when an observation is made on a microscopic particle, the observation process modifies the energy of the observed particle and some time is needed before the particle can return to its initial state.

In his formulation of quantum mechanics, Heisenberg derived a differential equation – now known as Schrodinger's equation - that could be used to solve for the wave function of a quantum system. Despite the success of his approach, Heisenberg himself tried restlessly to find the right interpretation of the wave function. The correct interpretation of the wave function– the most widely accepted one among physicists nowadays – was later formulated by the German physicist Max Born: In the case of a one-particle system, the square of the modulus of the wave function evaluated at a point in the space-time domain, gives the probability of locating the particle at that point (here we have assumed that the integral of the square of the modulus of the wave function at any time over the whole space is equal to one).

The success of quantum mechanics cannot be overstated. It served as an extension of Bohr's model to atoms having many electrons and was used to reproduce the results of Planck in the Blackbody radiation and Einstein in the Photoelectric effect. Erwin Schrodinger, Paul Dirac, and Werner Heisenberg were awarded the Nobel Prize in physics in 1932, 1933, and 1933 respectively for their work in quantum mechanics.

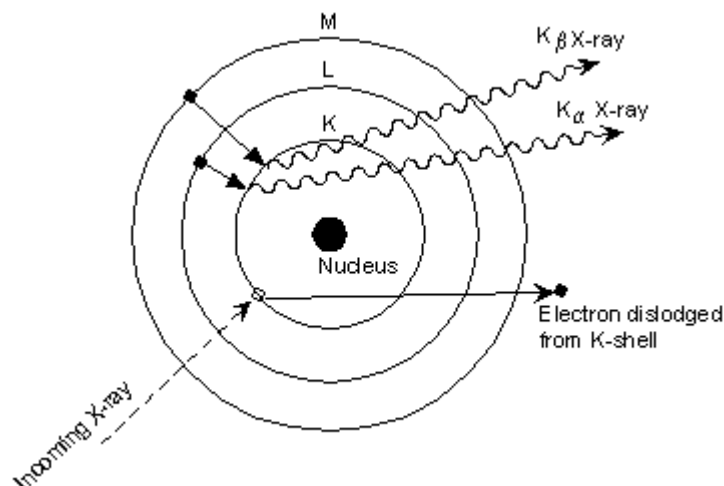


Figure 2-20: Paul Dirac

3 Modern applications of quantum physics

3.1 X-Rays

Often referred to as Rontgen Radiation, X-rays are a form of electromagnetic radiation (in the invisible part of the spectrum) with short wavelength (shorter than UV radiation, about 10⁻¹⁰ nanometers) and high energy (120eV-120keV). They were discovered by the Dutch physicist Wilhelm K. Roentgen. They are produced in evacuated glass tubes during the collision of high velocity electrons (accelerated by creating a large potential difference within the tube) with a metal target metal made of molybdenum, tungsten or platinum for example. During the collision, the accelerated electrons knock out of the atoms of the targeted metal, electrons from the inner shell (K shell). An electron from an outer shell then moves to fill in the vacancy in the K shell, emitting an X-ray photon in the process as depicted in Fig. 3-1. The process produces a discrete spectrum of X-ray frequencies called spectral lines. The spectral lines produced depend on the metal used and are thus called characteristic lines or characteristic X-rays. X-rays can be detected through the use of photographic plates, scintillators, Geiger counters etc...

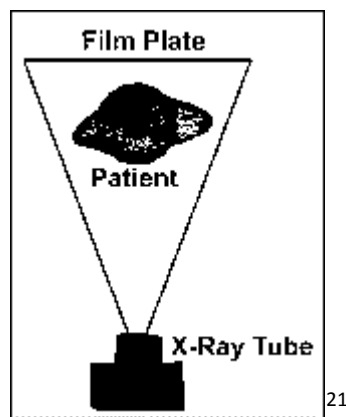


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Figure 3-1: Production of X-rays

Nowadays, X-rays are widely used in medicine for diagnostic radiography and in crystallography. They are mainly used in medicine to detect pathology of the skeletal system and to detect some disease processes in soft tissues (chest X-ray to identify lung diseases such as lung cancer or pneumonia, and abdominal X-rays). To develop a conventional medical X-ray, the patient is put on a film and a single burst of radiation is directed through the patient and onto the film as shown in Fig. 3-2. Since different parts of the body absorb X-rays differently (dense bones absorb more X-rays than soft tissues) a shadow like picture is formed. Even though X-rays are very useful, they have an inherent limitation: The picture that is obtained is a superposition of all the shadows that result from the radiation through layers of the body. Hence to determine which part of the X-ray corresponds to which layer of the body is difficult. This makes X-rays useless in the imaging of soft tissues such as brain and muscles.

Figure 3-2: Ordinary X-ray system



This inability of X-rays to image soft tissues has been improved through the use of techniques such as CAT (Computerized Axial Tomography) scan and CT (Computerized Tomography) scan, which make it possible to produce images of specific tissues of the body, which are not obscured by the other organs. In each one of these techniques a series of fanned out beam of X-rays are passed through the patient simultaneously and are collected on the other side by a detector which records the intensity of the beam. Different intensities correspond to different body tissues. In a CAT scan (see Fig. 3-3), the X-ray source is rotated to different orientation, and the data collected by the detectors are inverted by computers using mathematical formulas to produce a high resolution image of the cross sectional slice of the body (one that is perpendicular to the body's long axis hence the "axial" in CAT). CAT scans have helped to revolutionize the field of radiology, neurology and nuclear medicine.

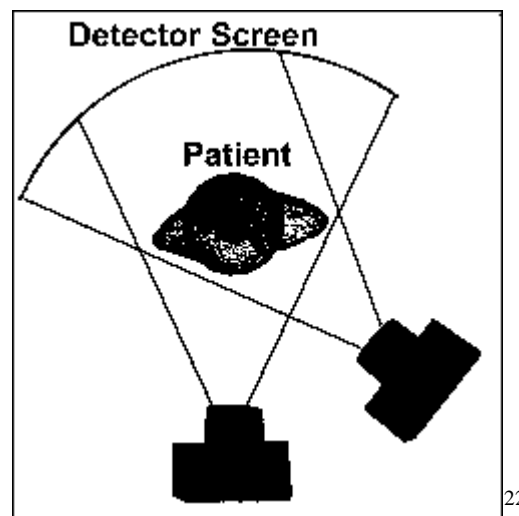


Figure 3-3: Schematic of a CAT scan.

Despite their usefulness, X-rays are a form of ionizing radiation and as such can be dangerous. They are classified by the U.S governments as Carcinogens. The measure of an X-ray ionizing ability is called the exposure (measured in coulomb per kilogram c/kg) and the effect of X-ray

ionizing radiation on matter is called the absorbed dose (measured in Joules per kilogram). Their ability to ionize body tissues is what makes them useful in the medical treatment known as radiotherapy used to manage the development of cancer. The dose of radiation applied in these treatments is usually higher than the one used for imaging. Lead can be used as a shield against X-ray radiation due to its high density, low cost and easy installation. The thickness of the required lead shield increases with the frequency of the X-ray radiation.

X-rays are also used for X-ray crystallography, X-ray astronomy, X-ray microscopic analysis, X-ray fluorescence and industrial radiography. X-ray crystallography is a method in which scattered X-rays are used to determine the arrangement of atoms within a crystal (see Fig. 3-4). Since a lot of materials can form crystals: minerals, salt, metals organic and inorganic biological molecules... X-ray crystallography is very useful in various fields of science. Using the mathematical theory of Fourier transforms, a crystallographer can reconstruct a three dimensional picture of the density distribution of electrons within the crystal using diffracted X-ray beams produced at different angles. The electron density distribution can then be used to determine the structure of the crystal, the type of bond within its atoms and many other properties. In its early days it was used to determine the size of atoms and the nature and length of the chemical bonds between them. It has also been used in biology to reveal the structure of nucleic acid such as DNA, RNA and to explain the functioning of biological molecules such as drugs, proteins and vitamins.

X-ray astronomy is the branch of astronomy which deals with the X-ray emissions of celestial objects. Since the earth's atmosphere absorbs most of the X-rays that are incident on the earth from outer space, X-rays measurements must be carried out at high altitudes. The celestial bodies producing cosmic X-rays usually contain very hot gases (in the magnitude of millions of Kelvin)

and are usually compact stars such as black holes and neutral stars. These X-rays are formed as hot gases are accelerated during their fall into celestial body having very high gravitational field. An X-ray microscope is a device that uses X-rays to generate images of very small objects. Unlike conventional microscopes where the image can be seen directly by an observer, X-rays produce images by reconstructing the data collected on an exposed film. This is due to the fact that X-rays have short wavelengths compared to visible light and cannot be easily reflected or refracted.

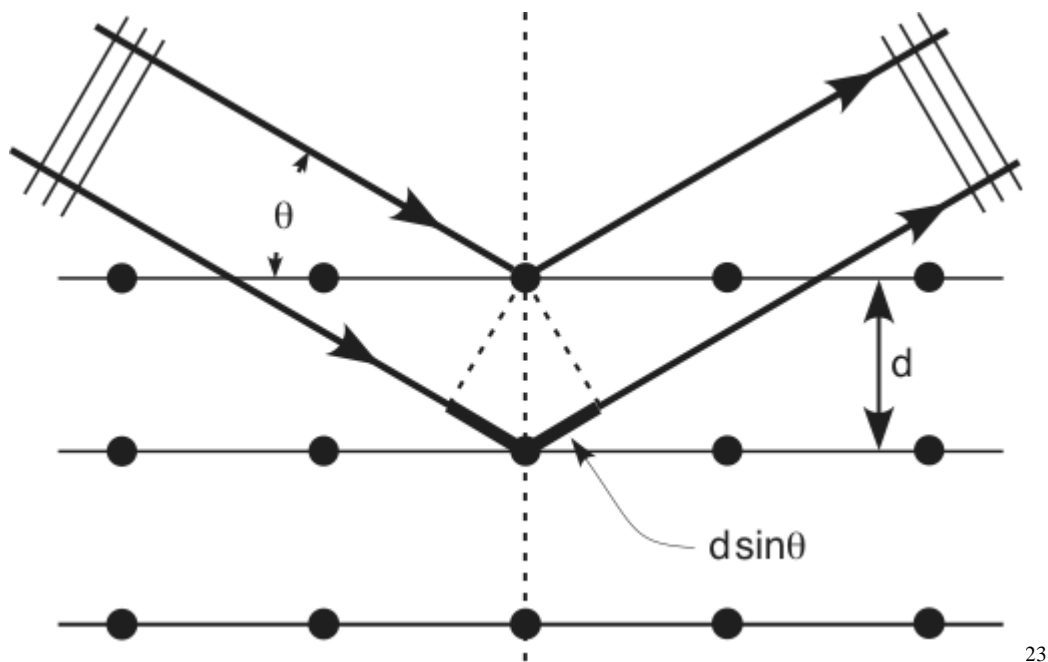


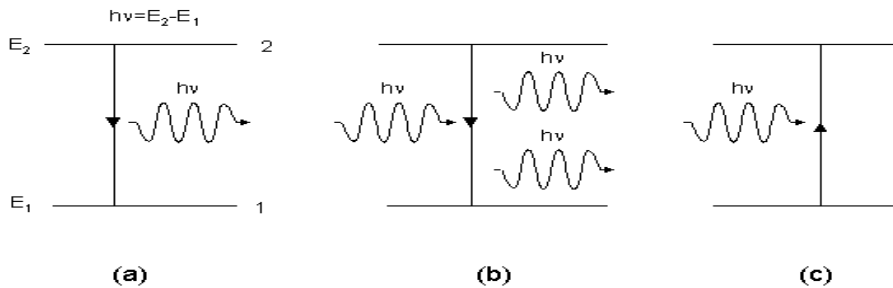
Figure 1: X-ray crystallography.

X-Ray fluorescence is the emission of secondary X-rays by a material that has been bombarded with high energy X-rays or gamma rays. The process occurs when a metal is exposed to X-rays with energy higher than its ionizing energy. X-rays knock off electrons from the inner shell making the atom unstable. An electron from an outer shell then falls into the inner shell to occupy the vacancy, emitting a photon with energy equal to the difference of energy of the two

orbitals involved. The material thus emits X-ray radiation with characteristic proper to that of the atoms used. It is this process that is used in Geiger counters to detect X-ray radiation.

3.2 LASERS

Lasers (Light amplification by the stimulated emission of radiation) are a device that emits light through a process called stimulated emission. Lasers are one of the most useful inventions of the 20th century. The principle behind their operation is based on the quantum mechanical picture of the atom. When an electron moves from a higher energy orbital to a lower one, a photon is emitted in the process. The emission process can be one of two types: stimulated emission or spontaneous emission. In a spontaneous emission, the photon is emitted spontaneously in a random direction without any outside excitation. In a stimulated emission, an incoming photon is used to stimulate the transition in energy levels. To do this the incoming photon must have the energy that matches the difference between the two energy levels ($E_{\text{photon}} = E_i - E_f$). The process is depicted in Fig. 2-5. One of the important features of stimulated emission is that every emission doubles the population of photons (hence the “amplified” in laser), and the emitted photon travels in the same direction as the incident photon. Furthermore the electromagnetic waves for the emitted and incident photons are coherent (same phase). To start a laser, an external source of energy is applied to excite the electrons of the atoms of a gas in a low pressure tube into higher energy levels (this is usually done by intense flashes of high voltage light and by high voltage discharges). When the voltage gradient is high enough, more electrons move from inner shell to higher energy orbitals. This process is called population inversion. In lasers the population inversion is used to create a metastable atom (an atom having a longer lifespan than ordinary excited electrons).



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Figure 3-5: a) spontaneous emission, b) stimulated emission, c) controlled emission.

The requirement for a metastable higher energy state is important to give enough time to the process to maintain population inversion and produce a continuous laser beam (as opposed to a pulsed one).

In an ordinary helium/neon laser, a high voltage is discharged through a low pressure gas tube composed of 85% neon and 15% helium to sustain the necessary population inversion. The process begins when an atom through simultaneous emission emits a photon in the direction parallel to the axis of the gas tube. This photon through stimulated emission will excite the electrons of another atom, producing two photons. The process then repeats itself and is sustained by using silvered mirrors at either end of the gas tube to reflect incident photons. The silvered mirrors are arranged to be perpendicular to the axis of the gas tube and one end is made partially silvered to allow some photons to escape from the gas tube and to form the laser beam (see Fig. 3-6). When all the photons have a single frequency, the laser beam is said to be monochromatic. By reducing the escape area of the emitted photons, a very narrow laser beam can be produced. Such a beam can be produced with very high intensity (power per unit area) by

confining the laser beam to a very narrow region. This ability to produce confined high intensity electromagnetic radiation is what makes them useful in a variety of applications. There are many other types of laser such as the ruby lasers, the argon-ion lasers, the dye lasers, carbon dioxide lasers etc...

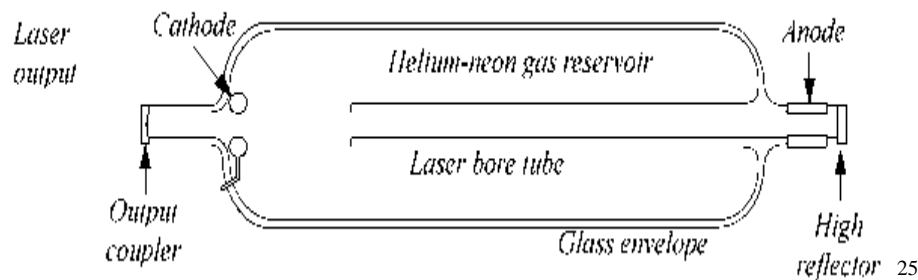


Figure 3-6: a schematic diagram of a helium/neon laser

One of the most interesting applications of lasers is in holography. Holography is a technique in which light scattered by an object is recorded and use to reconstruct a three dimensional image of the object. A hologram refers to the film on which the holographic image is recorded. To generate a hologram, a laser beam is shunned through a half-silvered mirror which reflects part and transmits part of the laser beam. The transmitted part is called the reference beam and the reflected part the object beam. The object beam is then reflected by the incident object and projected on a film (see Fig. 3-7). Since laser beams are coherent, an interference pattern (similar to the one observed in the double slit experiment) can be observed on the film, which is mainly composed of bright and dark fringes. The holographic image can then be produced by shining a laser on the interference pattern produced on the film as shown in Fig. 3-8.

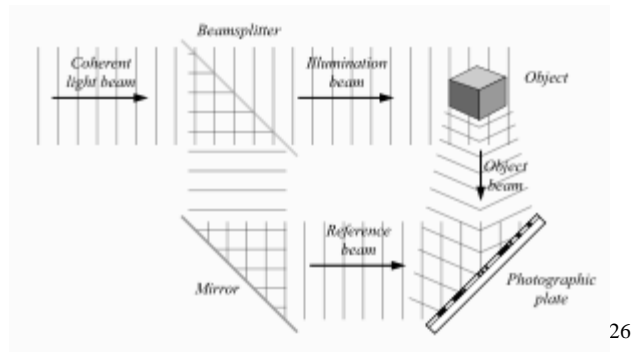


Figure 3-7: An arrangement used to produce a hologram

In fiber optic communication, lasers traveling within an optical wire are used to transmit information from one location to another. Lasers are also used to retrieve the data saved in optical storage devices such as DVDs and CDs by scanning their surface. In the metal industry lasers are used for welding, marking (by inscribing a certain pattern on a material's surface), cutting and bending. They are used in medicine with light activated drugs, in photodynamic therapy, in the treatment of cancer. In this procedure a drug is administered to the patient, and after absorption by the blood stream, is activated by a laser light in the region located near the cancer cell, creating a localized chemical reaction that disintegrates the cancer cells. Lasers are also used in medicine in the treatment of congenital capillary malformations (also known as port-wine stains) which affect 0.3% of children at birth. In this procedure, pulsed dye lasers are focused on the port-wine stain and their energy is absorbed by the oxyhemoglobin in the malformed capillaries which are destroyed in the process without damaging adjacent tissues. Lasers are also used in medicine, in ophthalmology, to correct nearsightedness and farsightedness. The procedure is known as photorefractive keratectomy (PRK). The conditions of near and farsightedness are due to the inability of the eye to properly refract light; hence producing images either in front or behind the retina. In PRK procedures this is corrected by

using lasers to burn and remove small amounts of tissues from the cornea of the eye with the intention to obtain a desired curvature. Lasers are used in the military for range determination, target designation and as direct-energy weapons (weapon that radiates a laser beam in an aimed direction).

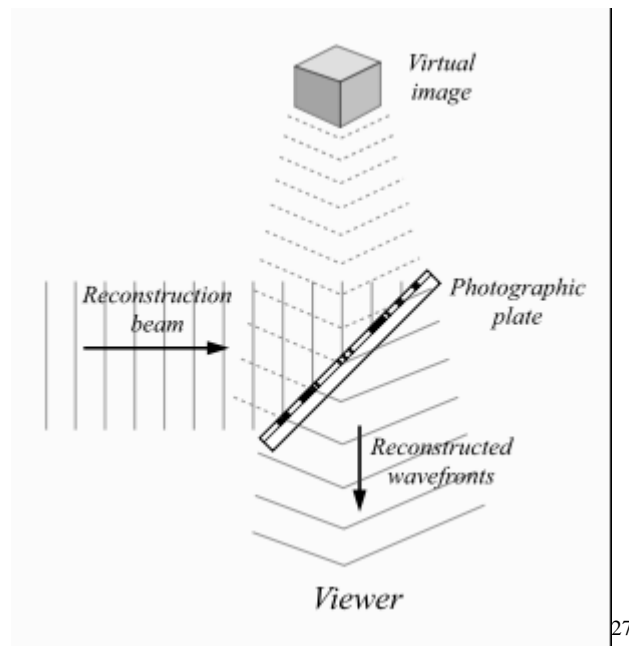


Figure 3-8: Holography reconstruction process

Despite their usefulness, lasers can be harmful. The exposure of a human eye to a laser that emits a wavelength that the cornea and lens can focus well, can be damaging as it can burn the retina by focusing a very intense light on a small spot which can cause permanent damage in a matter of seconds.

4 Conclusion

Although some people think of quantum physics as an abstract and purely academic subject, it has had a great impact on society through its applications. Without quantum physics, the great electronic revolution of the 20th and 21st century would not be possible. Quantum physics was able to solve the problems that classical physics faced at the beginning of the 20th century and with it we can explain the structure of atoms and molecules and how they interact with light. It is one of the most counter-intuitive theories of physics and is the product of the confluence of some of the greatest minds of our time.

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